MEASUREMENT OF THERMAL FLUXES IN POWER PLANT COMPONENTS

M. V. Stradomskii, E. A. Maksimov, and O. V. Fedorova

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A method of recovering the thermal flux acting on a sensing element with respect to measurements of sensing element signals is described.

Determination of nonstationary one-dimensional thermal fluxes based on temperature measurements at two different points of a component part along the flux constitutes an important practical problem. The solution of such problems is prompted by the need for information on the actual values of the energy density entering parts of various power plants, which is used as a basis for determining the direction of process control; moreover, measurement of the energy density may be treated as an independent problem. In the simplest case, the necessary information can obviously be obtained by means of two temperature sensors, for instance thermocouples, which are placed in a part at a certain distance from each other along the energy flow. However, difficulties arise in determining the coordinates of the temperature measurement point, and a priori determination of the direction of the energy flux vector is problematic. These errors can be reduced to a considerable extent by placing the thermocouples on a special part, which is usually referred to as the thermal flux data unit. This part, for instance, may consist of a disk with an assigned diameter-to-thickness ratio [1].

If the side surfaces are heat insulated, so that the thermal resistance in the axial direction is much lower than in the radial direction, the temperature field in the axial zone would be close to a uniform field if the energy flux through the heat-absorbing surface of the disk is uniform. In constructing the thermal flux data unit, high accuracy in determining the location of the thermocouple junction can be secured [2].

Figure 1 shows the schematic diagram of such a thermal flux data unit. It consists of a flat differential thermocouple with an intermediate thermoelectrode 1 and two external thermoelectrodes 2. The side surface of the differential thermocouple is covered with a heat-insulating material 3.

Measurements performed by means of such a data unit make it possible to calculate the thermal flux through its surface. If the heat loss through the side surface of the data unit is neglected, the temperature field can be determined by considering the model problem of heat propagation in an infinite plate, $-l < x < l$. If we know the temperature at one of the plate's boundaries and at some point inside it, we can determine the thermal flux density q(t) through the other plate boundary.

If the temperature at one of the plate boundaries is $T(l, t) = F(t)$, and the thermal flux through the other boundary is q(t), the thermal field inside the plate T(x, t) satisfies the thermal conductivity equation [3-5],

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2} \quad (-l < x < l,\ t > 0), \tag{1}$$

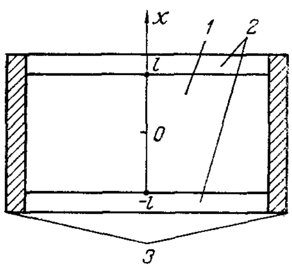

Fig. 1. Schematic diagram of the gradient thermal flux data unit.

Institute of Engineering Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev.
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for the initial condition

$$T(x,\ 0)=G(x) \tag{2}$$

and the boundary conditions

$$T(l,\ t)=F(t),\ -k\ \frac{\partial T(-l,\ t)}{\partial x}=q(t). \tag{3}$$

The boundary conditions (3) of the above boundary-value problem are nonhomogeneous. In order to use the Fourier method for solving this problem, we shall reduce it to a problem with zero boundary conditions [3]. We introduce the function $\Phi(y,\ \tau)$:

$$T(y,\ \tau)=\Phi(y,\ \tau)+F(\tau)+\frac{l}{k}q(\tau)(1-y), \tag{4}$$

which satisfies the equation

$$\frac{\partial\Phi(y,\ \tau)}{\partial\tau}=\frac{1}{\pi^2}\ \frac{\partial^2\Phi(y,\ \tau)}{\partial y^2}-\frac{\partial F(\tau)}{\partial\tau}-\frac{l}{k}(1-y)\frac{\partial q(\tau)}{\partial\tau} \tag{5}$$

and the following condition:

$$\Phi(y,\ 0)=G(y)-F(0)-\frac{l}{k}q(0)(1-y),\ \Phi(1,\ \tau)=0,\ \ \frac{\partial\Phi(-1,\ \tau)}{\partial y}=0. \tag{6}$$

Thus, using substitution (4), we obtain the nonhomogeneous equation of thermal conductivity (5) with zero boundary conditions (6).

The solution of the boundary-value problem (1)-(3) is given by

$$\begin{aligned}T(y,\ \tau)&=q(\tau)\ \frac{(1-y)}{k}\ l+F(\tau)+\frac{l}{k}\sum_{n=0}^{\infty}a_n(0)\exp(-\mu_n^2(\tau-\xi))(\cos\pi\mu_n y-\\&-(-1)^n\sin\pi\mu_n y)-\frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\sin\pi\mu_n}{\mu_n}\int_0^{\tau}d\xi\exp(-\mu_n^2(\tau-\xi))\times\\&\times\frac{\partial F(\xi)}{\partial\xi}(\cos\pi\mu_n y-(-1)^n\sin\pi\mu_n y)-\frac{1}{\pi^2}\frac{l}{k}\sum_{n=0}^{\infty}\frac{(-1)^n\sin\pi\mu_n}{\mu_n}\times\\&\times\int_0^{\tau}d\xi\exp(-\mu_n^2(\tau-\xi))(\cos\pi\mu_n y-(-1)^n\sin\pi\mu_n y),\ n=1,\ 2,\ 3\ldots\end{aligned} \tag{7}$$

Using the obtained analytical solution, we can calculate the temperature $T(y,\ \tau)$ at an arbitrary point y of the plate for the assigned initial and boundary conditions.

For the measured temperature values at the boundary and at a certain point y of the plate, this solution makes it possible to obtain a Volterra integral equation of the first kind with respect to the thermal flux:

$$q(\tau)=\Psi(\tau)+\int_0^{\tau}K(\tau-\xi)\frac{\partial q(\xi)}{\partial\xi}d\xi+\frac{k}{l}\int_0^{\tau}\mathcal{L}(\tau-\xi)\frac{\partial F(\xi)}{\partial\xi}d\xi, \tag{8}$$

where

$$K(\tau)=\frac{1}{(1-y)\pi^2}\sum_{n=0}^{\infty}\frac{\gamma_n}{\mu_n^2}\exp(-\mu_n^2(\tau-\xi))\{\cos\pi\mu_n y-(-1)^n\sin\pi\mu_n y\},$$

$$\mathcal{L}(\tau)=\frac{1}{(1-y)\pi}\sum_{n=0}^{\infty}\frac{\beta_n}{\mu_n}\exp(-\mu_n^2(\tau-\xi))\{\cos\pi\mu_n y-(-1)^n\sin\pi\mu_n y\},$$

$$\gamma_n=(-1)^n\sin\pi\mu_n,\ \beta_n=\sin\pi\mu_n,\ \Psi(\tau)=\frac{k}{(1-y)l}\{T(y,\ \tau)-F(\tau)\}.$$

The thermal flux $q(\tau)$ was calculated by means of the following recurrent formulas:

$$q(n\tau_1) = \Psi(n\tau_1) + \sum_{k=1}^{n} \nu_{k-1}\{S((n-k)\tau_1) - S((n-k+1)\tau_1)\} + \sum_{k=1}^{n} \eta_n \{Q((n-k)\tau_1) - Q((n-k+1)\tau_1)\},$$

$$\nu_n = \frac{q(n\tau_1) - q((n-1)\tau_1)}{\tau_1}, \tag{9}$$

$$S(\tau) = \int_0^\tau K(\tau-\xi)\,d\xi = \frac{1}{\pi^2}\sum_{n=0}^{\infty}\frac{\gamma_n}{\mu_n^4}\exp(-\mu_n^2\tau),$$

$$Q(\tau) = \int_0^\tau \mathcal{L}(\tau-\xi)\,d\xi = \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{\beta_n}{\mu_n^3}\exp(-\mu_n^2\tau),$$

$$\eta_n = \frac{k}{l}\,\frac{F(n\tau_1) - F((n-1)\tau_1)}{\tau_1}.$$

It should be noted that the above recurrent relationships hold if the variation of $\partial q(\tau)/\partial\tau$ during a time of the order of τ_1 is negligible.

It is most often found in practice that the temperature difference at the plate surfaces is much smaller than the absolute values of these temperatures. If the above problem is to be solved in terms of the temperatures proper, it would be necessary to perform operations with relatively large numbers, while only differences between them affect the calculation results. In such cases, it would be advisable to operate only with excess temperatures, i.e., $T_1(\tau) - T_0$ and $T_2(\tau) - T_0$, and determine these quantities in experiments.

Let us estimate the effect of the initial conditions on the temperature inside the plate. The initial conditions are comprised in the coefficient $a_n(0)$. The characteristic time of damping of the initial conditions is $t_0 = 1/\lambda_0^2 = (4l/a\pi)^2$. For our problem, $k = 25$ W/m·deg, $\rho = 8.9 \cdot 10^3$ kg/m^3, $\gamma = 410$ J/kg·deg. For $t \gg t_0$, the effect of the initial conditions can be neglected.

The accuracy of the results is affected by errors in assigning the data unit's dimensions and the location of the junction point and the errors in temperature measurements.

Assume that δl is the error in measuring the plate thickness and δx is the error of the junction location coordinate;

$$\alpha = \frac{\int_0^\tau d\xi K_1(\tau-\xi)\frac{\partial q(\xi)}{\partial\xi} + \frac{k}{l}\int_0^\tau \mathcal{L}_1(\tau-\xi)\frac{\partial F(\xi)}{\partial\xi}\,d\xi}{q(\tau)},$$

$$\beta = -\frac{x}{l}\alpha - \frac{\frac{k}{l}[T(y,\tau) - F(\tau)]}{q(\tau)} - \frac{\frac{k}{l}\int_0^\tau \mathcal{L}(\tau-\xi)\frac{\partial F(\xi)}{\partial\xi}\,d\xi}{q(\tau)},$$

and the relative error in the flux value is then given by

$$\frac{\delta q(\tau)}{q(\tau)} \sim \alpha\frac{\delta x}{l} + \beta\frac{\delta l}{l} + \frac{1}{1-y}\,\frac{\delta x}{l} + \frac{1}{1-y}\,\frac{x}{l}\,\frac{\delta l}{l}, \tag{10}$$

$$\frac{k}{l}(T(y,\tau) - F(\tau)) \sim q(\tau)(1-y),$$

$$\frac{k}{l}\int_0^\tau \mathcal{L}(\tau-\xi)\frac{\partial F(\xi)}{\partial\xi}\,d\xi \sim q(\tau)(1-y),$$

$$\int_0^\tau K_1(\tau-\xi)\frac{\partial q(\xi)}{\partial\xi}\,d\xi + \frac{k}{l}\int_0^\tau \mathcal{L}_1(\tau-\xi)\frac{\partial F}{\partial\xi}\,d\xi \sim q(\tau)(1-y),$$

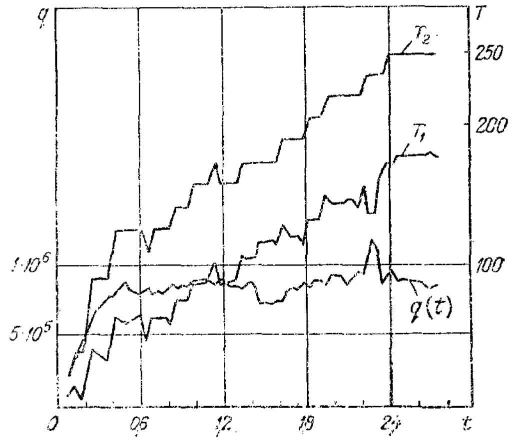

Fig. 2. Dynamics of temperatures at the sensing element surfaces and of the thermal flux recovered on the basis of these temperatures.

where α and β are approximately equal to unity. Hence it follows that, for y values close to 1, the relative error in calculating the flux exceeds considerably the error in measuring the data unit's dimensions.

The flux error caused by inaccuracies in temperature measurements is given by

$$\delta q \sim \frac{k}{l}\ \frac{1+nf_{\mathcal{L}}}{1-nf_k}\,\delta T,$$

where δT is the temperature error, and $n \sim 1/\tau_1$; f_L and f_k are of the same order, so that the coefficient in front of δT is equal to about 1, whence it follows that the flux errors are of the same order as the errors in assigning the excess temperature.

Our analysis shows that the farther the temperature measurement point from the heat-absorbing surface, the smaller the temperature fluctuation amplitude (for the same amplitude of the thermal flux density and its variation frequency) and, consequently, the larger the relative error in determining (assigning) this temperature in experiments which, naturally, increases the error of recovery of the thermal flux density.

Moreover, data on the thermal flux density cannot be obtained directly at any instant of time, since the data unit's temperature reaction to a change in the energy density at its boundary lags in time behind this change, the more so the farther the temperature measurement point from the surface, i.e., the temperature field of the plate obtained with respect to the temperatures measured at a given instant of time can be used to determine the energy density at the boundary that had occurred somewhat earlier.

The inertia of the material layer above the temperature measurement point can often be neglected ($\delta \to 0$), in which case the temperature measurement point virtually lies on the heat-absorbing surface. Such an approximation is ordinarily used for determining the density of thermal fluxes that vary relatively slowly.

As an example, Fig. 2 shows the dynamics of temperatures at the sensing element surfaces in a thermal flux data unit during the startup from cold of a power plant. These temperatures have been calculated with respect to one of the temperatures and the temperature difference between the surfaces, measured by means of the data unit. The figure also shows the variation in time of the thermal flux density, calculated by means of the proposed method.

In this case, the error in finding the thermal flux density is determined by the error in measuring the temperature and the frequency of temperature measurements [6]. Since the temperature measurements are performed with a frequency of 25 Hz, all information at frequencies above 12.5 Hz is filtered out, and we can only recover thermal flux densities at frequencies below 12.5 Hz. The error in recovering these densities and the temperature measurement error are of the same order of magnitude.

NOTATION

k, thermal conductivity coefficient; a^2, thermal diffusivity coefficient; γ, specific heat; ρ, density; $\tau = (a\pi/l)^2 t$, dimensionless time; $y = x/l$, dimensionless coordinate; $T_1(t)$, temperature at the top boundary; $T_2(t)$, temperature of the heat-absorbing surface; T_0, reference temperature; t, time (sec); q, thermal flux density (W/m^2); T, temperature (°C).

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